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CROSS-LATTICE-STRUCTURE-BASED MODELING OFTHE HUMAN SPINAL COLUMN

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ABSTRACT

This paper reviews 2-D and 3-D models of the human spinal column. 3-D static modeling of the human spinal column is of prime importance in posture study and training as well as management of spinal deformities, in particular, scoliosis, kyphosis and lordosis. 3-D dynamic modeling is useful in studying and interpreting human gait. In this work, crystal-structure-based model of the human spinal column, proposed in 2012, has been reformulated using the concept of cross lattice proposed earlier. Cross lattice is a dimensionless lattice, whose base vectors are constructed by taking cross product of direct lattice and reciprocal lattice. Such a model should help better understand the human spinal column and could be utilized to develop efficient and effective management strategies for deformities of spine.

KEYWORDS: Moiré fringe topography, Dotted rasterstereography, Scoliosis, Kyphosis, Lordosis, Crystal structure

INTRODUCTION

Anthromathematics (mathematics of body sizes, forms, proportions and structures) of the human spinal column is going to be one of the most active research areas in the twenty-first century, which should include modeling of the human spine based on ideas from biomechanics, computer science, mathematics and physics, so that one improves the understanding of the etiology and the prognosis of spinal deformities in a growing child.

This paper reviews two-dimensional and three-dimensional (static and dynamic) models of the human spinal column and describes modeling based on cross-lattice structure, proposed earlier by the author to handle problems in condensed-matter physics. Cross lattice is a lattice structure, whose base vectors are obtained by taking cross product of direct lattice vectors and reciprocal lattice vectors. This operation makes these vectors dimensionless. This model is an enhancement of the crystal-structure-based model of the human spinal column, proposed in 2012.

SPINAL DEFORMITIES IN A GROWING CHILD

The most significant spinal deformities in a growing child are scoliosis, kyphosis and lordosis. Scoliosis is defined as lateral curvatures and rotations of the spinal column. Kyphosis and lordosis are accentuated curvatures of the spinal column as viewed in sagittal-plane projection. An enhanced curvature in the thoracic region would be called kyphosis, a reduced one termed as flat back. An increase in curvature in the lumbar

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[§]*The NGDS Pilot Project URL***:** https://ngds-ku.org

region should be attributed to presence of lordosis. Frequently, as a reaction to thoracic-region increased curvature there is a compensatory enhancement produced in the lumbar region. Such a condition is recognized as kypho-lordosis.

Out of these three, scoliosis is the most common and poses a real challenge of the health and the well-being of growing children. If left untreated, this condition may disfigure the body and limits functionality of a youngster so that such an individual may not be gainfully employed in many professions (Kamal *et al.*, 1998). Even housewives are not able to perform essential daily chores and may even effect their capability to bear children and care for young offspring. Hence, it is of utmost important to implement scoliosis screening programs in schools and community health centers supported by an active group involved in modeling of scoliotic spine (Horn, 2012). Scoliosis Research Society recommends scoliosis screening as valuable from the points-of-view of technical efficiency as well as clinical, program and treatment effectiveness (Labelle *et al.*, 2013). Earlier, Luk *et al*. (2010) concluded on the basis of a large study that school scoliosis study in Hong Kong is productive and sensitive with a low referral rate. Saikia *et al.*, (2002) conducted an epidemiological study for presence of scoliosis in this part of the world. In order to get fruitful results educational programs for parents as well as children should be conducted (Kamal, 1997*b*; 2017).

QUANTIFICATION OF RISK OF ACQUIRING SCOLIOSIS

Orthopedic surgeons feel a dire need to identify factors associated with scoliosis in school children (Baroni *et al.*, 2015). An *et al.* (2015) conducted prevalence study of adolescent idiopathic spine in 10-11-*year* old children. Power of mathematics was utilized to develop such indices, which could indicate risk of scoliosis in prepubertal, peripubertal and pubertal children[1]. These indices go much deeper than asymmetry about the sagittal plane indicated by moiré fringe topography (Akram and Kamal, 1991; Kamal, 1997*a*; Kamal and El-Sayyad, 1981; Kamal and Lindseth, 1980; Kamal *et al.*, 2013*c*; 2014), rasterstereography (Kamal *et al.*, 2013*a*) and dotted rasterstereography (Wasim *et al.*, 2013) and could be used in deciding which child should be put under observation. Being highly sensitive, moiré screening generate a large number of cases to be followed up, which overburdens the health-care resources, resulting in non-availability of essential medical surveillance to those, who are at the highest risk of monitoring.

Cumulative-Scoliosis-Risk Weightage (CSRW): CSRW was proposed by our group 6-*year* ago (Kamal *et al.*, 2013*e*), which allocated a weight to each early-warning signal of scoliosis, which incorporated, family history (risk is increased if scoliosis detected in parents/siblings), age slot (3 *years* to less than 6.5 *years*; 6.5 *years* to less than 7.5 *years*; 7.5 *years* to less than 8.5 *years*; 8.5 *years* to less than 11 *years*), degree of tallness (more than 10^P; 20^P; 30^P — superscript P stands for percentile), degree of wasting implying lesser mass-for-height (more than 10%,; 20%; 30%), positive forward-bending tests (indicating lumbar or thoracic asymmetry), nonalignment of plumb-line, positive indicators in visual examination of back (C or S shape of midline of back; shoulders drooping; scapulae uneven; body triangles unequal; spinal dimples uneven) and positive moiré (back; front), with the weightage increasing if the condition was present for more than one checkup. Various tests to examine spinal column are described in detail in a previous publication (Kamal *et al.*, 2015).

The shortcoming of CSRW is because of the fact that if some tests are not conducted/information not available on a set of students (*e. g.*, history information), this index may not be compared with other pupils in that class.

Normalized-Scoliosis-Risk Weightage (NSRW): NSRW was proposed 3-*year* ago as a percentage (Kamal *et al.*, 2016; Raza, 2016). It is computed by multiplying CSRW with 100 and dividing by the sum of maximum values of scores of individual items, corresponding to a certain checkup.

$$NSRW = 100 \frac{CSRW}{\sum score_{\max}} \% \qquad (1)$$

This definition should be able to incorporate additional information available by inclusion of further tests in later sessions, so that the enhanced index may be compared for data collected in different years.

Integrated-Trunk-Deformities-Screening Protocol: 23-*year* ago, Kamal *et al.* (1996) put forward this protocol, which consisted of multiple-level screening of primary-school pupils. The checks included at the top level were designed to be highly sensitive. The checks located at the bottom level were selected to be highly specific[2]. The objective was to reduce X-ray exposure to youngsters, while pinpointing at-risk cases for neuro-orthopedic evaluation (Cottalorda *et al.*, 2012). Although the protocols seemed to be useful initially, the number of tests needed and the special environment required did not make this protocol implementable for mass screening.

SCOLIOSIS CASE-FINDING AND RULING OUT SCOLIOSIS-LIKE CONDITIONS

Scoliosis, a condition which may disfigure the body severely and affect vital organs including heart, lung as well as stomach, should be discovered through a 2-*minute* compulsory orthopedic examination with the child bareheaded, barefooted and completely undressed except short underpants (Linker, 2012). Protocols of such examina-tion are explained in detail elsewhere (Kamal *et al*., 2015). For scoliosis case-finding, early-warning signals in the visual inspection of back of stripped child in the attention position are meticulously observed and noted (Fig. 1). Visual examinations should be accompanied by plumb line, engineering T and graph screen superimposed on back photograph. These tests end up with many false positives in pupils studying in primary school (Kamal *et al.*, 2013). These students are then required to get AP-X ray of the entire spinal column from external auditory meatus to hip joint to rule out or confirm entennnnntireireion attention

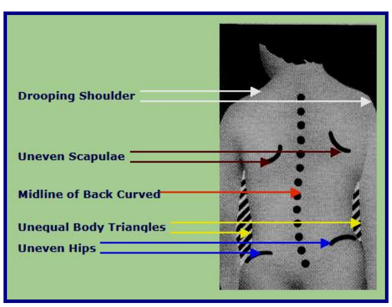

Fig. 1. Visual cues employed in scoliosis case-finding — body asymmetry about sagittal plane a warning sign.

scoliosis (AP stands for anteroposterior). X rays consist of ionizing radiations, which damage the delicate bone marrow of the growing youngster (Kotwicki, 2008). Hence, it is of utmost importance that radiation techniques are used sensibly to minimize radiation burden (Cassar-Pullicino and Eisenstein, 2002).

In order to avoid over-treatment (Kamal *et al.*, 2013*b*; 2016), it is of utmost importance to rule out scoliosis-like conditions, which frequently display artificial scoliosis (Kamal *et al.*, 2013*d*). Examples include problems of posture, leg lengths not equal and weakness of hip(s). In order to highlight these factors, two-level differential-spinal-function testing (DSFT) is conducted, explained with photographs and flowcharts elsewhere (Kamal *et al*., 2015). In the first level, two tests are conducted and the results interpreted 'to suspect' a possible condition. In the second level, a third test is performed 'to indicate' the suspected condition (Kamal *et al*., 2014*a*; 2015; Sarwar, 2015). At the end of DFST, the screener should be able to form an opinion regarding existence of true lateral curvatures and true spinal rotations.

MONITORING THE SCOLIOTIC CURVE

Photography should be employed to document detected cases of scoliosis, which should, subsequently, be quantified through moiré fringe topography (Kamal *et al.*, 2013*c*; 2014*a*), rasterstereography (Kamal *et al.*, 2013*a*) and dotted-rasterstereography (Wasim *et al.*, 2013). The above-mentioned techniques use radiations that do not penetrate the body. Further, these radiations are non-ionizing. Therefore, they are harmless for the body of a growing youngster. X rays should be taken at the first visit and after that only when a change is seen in moiré and raster patterns (Kamal and Lindseth, 1980). In 2013, the author proposed 3-D-spinal-column surface analysis, providing height and curvature maps, by combining moiré fringe topography and rasterstereography with backscatter-X-ray-scanning technology. This technique has the potential to generate, not only, the out-line, but also, the structure of spinal column using low dose X rays used in full body scanning (Kamal, 2013*a*; *b*).

Although Ferguson and Tideström methods have been mentioned in orthopedic texts,

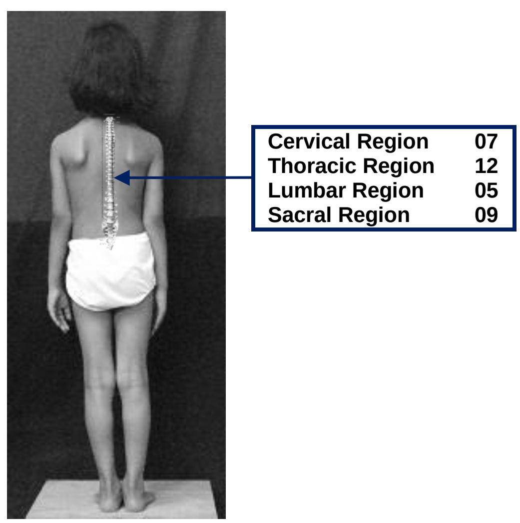

Fig. 2. Anatomy of the human spinal column — distribution of vertebrae in different regions.

Scoliosis Research Society recommends using only the Cobb method of quantifying the scoliotic curve (Neugebauer, 1983).

The author defined *Asr angle*[3] in the context of 3-D modeling of the spinal column (Kamal, 1982*a*; 1983*a*; *b*). Asr Angle is constructed through parallel transport of skewed lines, one of the lines is drawn normal to the plane, which is tangent to the superior endplate of one vertebra above the fracture and another drawn normal to the plane, which is tangent to the inferior endplate of the vertebra one level below the fracture.

MODELING OF THE HUMAN SPINAL COLUMN

The human spinal column consists of 33 vertebrae — top 7 constitute the cervical region, the next 12 form the thoracic region, another next 5 make up the lumbar region and the last 9 define the sacral region (Fig. 2).

Two-dimensional models: 2-D models are based on frontal view from projections of the spinal column obtained from AP-X rays or back moiré topographs, both obtained in the attention position (Oxborrow, 2000). The author attempted to obtain Cobb angles from moiré pictures of back (Kamal, 1982*b*; El-Sayyad and Kamal, 1981).

Three-dimensional models: Spinal column is, in fact, a 3-D structure. AP-X-ray pictures are not suitable for properly visualizing kyphosis or lordosis. 3-D-spinal-column models have the capability to generate full view from projections of spinal column in the sagittal and the frontal planes, synthesized from lateral and AP-X-ray pictures. On the other hand, only one back moiré topograph in the attention position was able to generate both views (Kamal 1982*a*). A few years ago, Bella *et al.* (2014) have defined spinal shape using moiré method.

3-D-static models were formulated simultaneously in Germany (Hierholzer and Lüxmann, 1982) as well as in United States (Kamal, 1982*b*; 1983*a*; *b*). The 1982-83 model proposed by the author were based on line segments forming the angle of spinal curvature. Later the model was formulated in terms of curvatures (Kamal, 1987). Included later on were the natural curvatures of the spine, visible in the lateral projection (Kamal, 1996*a*). From X-ray or moiré measurements, a curve was generated, relating x, y and z where (Kamal *et al.*, 2016)

$$x = x(\xi),\; y = y(\xi),\; z = z(\xi) \qquad \text{(2a-c)}$$

as a best fit to discrete measurements obtained at various locations represented by the parameters ξ_i; i = 1,…,33; corresponding to 33 vertebrae of the spinal column. The parameters ξ_i were representing lengths along spinal column with origin at the level of external auditory meatus, the length increasing towards hip joint. In the neighborhood of a certain point on the spinal column, this curve could be written as

$$x = f(y,z) = \frac{1}{2}ay^2 + byz + \frac{1}{2}cz^2 \qquad \text{(3)}$$

where $a = a(\xi_i), b = b(\xi_i), c = c(\xi_i)$, values obtained by solving simultaneous equations generated using equation (3) for three neighboring values of (x, y, z). For this purpose equation (3) was expressed as

$$x(\delta_i) = \frac{1}{2}a(\xi_i)y^2(\delta_i) + b(\xi_i)y(\delta_i)z(\delta_i) + \frac{1}{2}c(\xi_i)z^2(\delta_i) \qquad \text{(4a-c)}$$

where $\delta_i = \xi_i - \Delta$ in equation (4*a*), $\delta_i = \xi_i$ in equation (4*b*) and $\delta_i = \xi_i + \Delta$ in equation (4*c*), $\Delta << \xi_i$, assuming that values of *a*, *b* and *c* were same for these closely-located points. The cross term (yz) vanished, when the coördinate mesh[4] was rotated clockwise about the *x* axis through an angle α

(5*a*, *b*) $$y = y_{rot} \cos\alpha + z_{rot} \sin\alpha,\ z = -y_{rot} \sin\alpha + z_{rot} \cos\alpha$$

where

(6) $$\alpha = \frac{1}{2} \tan^{-1} \frac{2b}{c-a}$$

The curvatures were computed from the coefficients of squares of rotated coördinates, *i..e.*, ${y_{rot}}^2$ and ${z_{rot}}^2$

(7) $$x = f_{rot}(y_{rot}, z_{rot}) = \frac{1}{2} \kappa_{1i} {y_{rot}}^2 + \frac{1}{2} \kappa_{2i} {z_{rot}}^2$$

where

(8*a*, *b*) $$\kappa_{1i} = a + c - \frac{2b^2}{4b^2 + (c-a)^2},\ \kappa_{2i} = a + c + \frac{2b^2}{4b^2 + (c-a)^2}$$

The natural curvatures in a normal youngster (matched by gender and age, in whom no spinal deformities are discovered) were written as

(9) $$x = F_{rot}(y_{rot}, z_{rot}) = \frac{1}{2} K_{1i} {y_{rot}}^2 + \frac{1}{2} K_{2i} {z_{rot}}^2$$

where K_{1i} and K_{2i} represented natural curvatures of a normal child. Next, the child was asked to hang freely from a bar and any reduction in the deformity observed. The curvatures were, again, calculated after guarded-graduated[5] passive correction κ'_{1i} and κ'_{2i}. If K'_{1i} and K'_{2i} represented curvatures of the normal child in the hanging position, 'Degree of Correction of Spinal Deformity', *D*, was defined as

(10) $$D = \frac{50}{n} \sum_{i=1}^{33} \left[\frac{(\kappa_{1i} - \kappa'_{1i})^2}{(\kappa_{1i} - K'_{1i})^2} + \frac{(\kappa_{2i} - \kappa'_{2i})^2}{(\kappa_{2i} - K'_{2i})^2} \right] \%$$

Geometrically, if $\kappa_{1i} = \kappa'_{1i}$ and $\kappa_{2i} = \kappa'_{2i}$, there is no correction and $D = 0$. On the other hand, if $\kappa'_{1i} = K'_{1i}$ and $\kappa'_{2i} = K'_{2i}$, the deformity is completely corrected and $D = 100\%$. The deformity is classified as 'severe' $\left(0 \le D < \frac{100}{3}\%\right)$, 'intermediate' $\left(\frac{100}{3}\% \le D < \frac{200}{3}\%\right)$ or 'mild' $\left(\frac{200}{3}\% \le D \le 100\%\right)$. Table 1 shows classification of *D* and lists recommended treatment in each category. This 3-D-static model was found to be useful in the study of posture of children.

During same year, the author put forward 3-D-dynamic model as a generalization of the 3-D-static model, to study movement of the human spinal column during a gait cycle (Kamal, 1996*b*).

Crystal-structure-based model: The human spinal column is a set of vertebrae in the cervical, the thoracic, the lumbar and the sacral regions, located at certain distances from each other. Crystal-structure analogy was applied to model the spinal column. The center-of-mass of each vertebra was expressed in terms of positional coördinates (x, y, z) in the body-coördinate system. From the perspective of crystallography, this could be interpreted as *form factor*. Adding rotational (in terms of Euler angles) and inter-vertebral-spacing information, the analysis takes the shape of *structure factor*, used in solid-state physics to study crystal structure (Kamal *et al.*, 2012*b*; 2014*b*).

Other models: Nissan and Gilad (1984) proposed an anthropometric model for the cervical and the lumbar vertebrae. Punjabi *et al.* (1989) studied spinal stability and intersegmental muscle forces in the context of a biomechanical model. Monheit and Badler (1990) put forward a kinematic model of the human spine and torso. Dietrich *et al.* (1991) developed a biomechanical model of the human spinal column. Yoganandnan *et al.* (1996) performed finite-element modeling C4-C6 cervical spine unit. Reeves and Cholewicki (2003) modeled human lumbar spine for assessing spinal loads. Faizan *et al.* (2012) conducted a finite-element-based study to figure out adjacent-level effects of bi-level disc replacement, bi-level fusion and disc-replacement plus fusion in cervical spine. Gruescu *et al.* (2015) modeled spinal column and simulated spinal deformities.

CROSS-LATTICE FORMULATION

Reciprocal lattice is, widely, used in condensed-matter physics and materials science. The expressions of reciprocal-lattice vectors, having dimensions of inverse of length, are changed, when different systems of units (Bukhari and Kamal, 1987) are used. In order to alleviate this problem, a modified lattice, termed as *cross lattice*, was introduced (Kamal and Husain, 1978; 1988; Kamal, 2011) and later applied to liquid-crystal structure (Kamal *et al.*, 2012*a*)[6].

The cross-lattice vectors were constructed in such a way that they became independent of the coördinate system chosen. A coördinate-independent representation is a pre-requisite of representing generalized laws and forms the basis of principle of general covariance (Kamal, 2009*b*). This was the motivation to present the most general coördinate transformation. Let $\hat{\boldsymbol{a}}_1$, $\hat{\boldsymbol{a}}_2$, $\hat{\boldsymbol{a}}_3$, be the basis vectors for ordinarty lattice. Any

toooooor may be represented in

Table 1. Severity of 'Degree of Correction of Spinal Deformity' (*D*) and recommended treatment

Range of D	*Severity Level*	*Recommended Treatment*
0 - 33.33 %	**Severe**	**Surgery**
33.34 % - 66.66 %	**Intermediate**	**To be decided by the orthopedic surgeon**[£]
66.67 % - 100 %	**Mild**	**Combination of exercises and brace**

[£]The decision should depend on the location and the progression of scoliotic curve as well as the numerical value of degree of correction of spinal deformity (D) — how close the value is to 33.33% (inclination towards surgical treatment) or 66.66% (inclination towards a combination of exercises and brace) vector $\boldsymbol{L}$, may be represented in terms of these linearly-independent bases $\boldsymbol{L} = L_k \hat{\boldsymbol{a}}_k$ (repeated indices denote summation according to Einstein convention; all latin indices take up the values 1, 2, 3). The recipriocal-latttice vectors for a non-rectangular lattice, may be constructed, using the basis

$$\hat{\boldsymbol{g}}_k = \in_{klm} \frac{\hat{\boldsymbol{a}}_l \times \hat{\boldsymbol{a}}_m}{2\hat{\boldsymbol{a}}_1 \bullet \hat{\boldsymbol{a}}_2 \times \hat{\boldsymbol{a}}_3} \qquad (11)$$

as $\boldsymbol{G} = G_k \hat{\boldsymbol{g}}_k = 2\pi\, n_k \hat{\boldsymbol{g}}_k$; the numbers n_k are integers, $\in_{klm}$ is Levi-Civita symbol[7], whose values are +1, when the indices form even permulation (123, 231, 312), –1, when the indices form odd permulation (132, 213, 321) and zero otherwise (one or more indices repeated). Equation (11) is obtained by applying the periodicity condition $f(r+L) = f(r)$. The reciprocal lattice vectors satisfy the orthonaormality condition $\hat{\boldsymbol{a}}_i \bullet \hat{\boldsymbol{g}}_j = \delta_{ij}$; where δ_{ij} is Knoecker delta, whose value is 1 when $i = j$ and zero otherwise. A cross product of the direct- and the rciprocal-lattice vectors is dimensioneless, and named as *cross lattice*, $\hat{\boldsymbol{e}}_i = \hat{\boldsymbol{a}}_i \times \hat{\boldsymbol{g}}_i$ (no summation over i). The cross-lattice length can be written as

$$\boldsymbol{D} = D_k \hat{\boldsymbol{e}}_k = 2\pi\, n_k \hat{\boldsymbol{e}}_k \qquad (12)$$

The following results can, easily, be derived from the above deinition

$$(13a\text{-}d)\quad \boldsymbol{D} \bullet \boldsymbol{L} = \boldsymbol{D} \bullet \boldsymbol{G} = 0;\ \exp(i\boldsymbol{D} \bullet \boldsymbol{L}) = \exp(i\boldsymbol{D} \bullet \boldsymbol{G}) = 1,\ (i = \sqrt{-1})$$

The eigenfunctions of cross lattice can be obtained by considring an anisotropic potential, which depends on angle, θ. The band must be well separated to make non-diagonal matrix elements vanish with respect to band index. Defining a modified Brillouin zone to be a Wigner-Seitz cell in cross lattice and letting $\boldsymbol{D}$ take all values inside this zone, all the coefficients vary and become functions of $\boldsymbol{D}$. However, because of the periodicity in the momentum space, they are all identical functions of $\boldsymbol{D}$, translated in cross-lattice space.

MODELING OF SPINAL COLUMN BASED ON CROSS LATTICE

To model the spinal column based on cross-lattice formulation, the center-of-mass of each vertebra was expressed in terms of cross-lattice lengths (D_1, D_2, D_3) in the cross-lattice-coördinate mesh. *From factor* and *structure factor* were defined accordingly for cross-lattice-structure-based model.

Applications: Wannier functions are useful for systems, which are *isotroptic*, but *inhomogeneous*. The cross eigenfunctions, belonging to eigenstates of angular momentum in cross-lattice representation, are useful for systems, which are *homogeneous*, but *anisotropic*, like the human spinal cord.

CONCLUSION

Spinal column is one of the most important organs of the human body. If damaged it may inflict morbidity and in severe cases even mortality. Therefore, it becomes essential to model spinal column and spinal deformities so that effective management strategies be devised. In this paper different models of spinal column have been discussed and cross-lattice-structure-based model proposed, which has the prospects of being applied in simulation of spinal deformities.

DEDICATION

This paper is dedicated to the loving memory of my most respected teacher and mentor Professor Mohummed Idrees Khan (*born*: Wednesday, April 2, 1941, District Jaipur, Rajhistan, now in India; *passed away*: Friday, December 21, 2018, Hyderabad, Sindh, Pakistan). He taught the author subject of physics in Government College, Hyderabad in the year 1973, a college in which he acted as Principal during 1997. He got his early education (classes 1-9) from Shahdadkot, Qambar Shahdadkot District, Sindh, Pakistan and completed his matriculation from Government High School, Nowshera, KP, Pakistan. He completed his HSC and BSc from Government College, Hyderabad and MSc from University of Sindh, Jamshoro, Sindh. In 1998 he was appointed Director, College Education, Hyderabad Region, a position on which he worked till his retirement in 2001. He was instrumental in getting his alma mater upgraded to Government College University.

The revered teacher belonged to Qaim Khani community, which is very active in welfare work in Hyderabad. He was Treasurer/Secretary of Hilal-é-Ahmer and served as Chairman of Ushr-and-Zakat Committee in Hyderabad, Sindh.

An excellent teacher and communicator, the late professor instilled in the author the spirit of inquiry and desire to achieve excellence in the subject, keeping in focus the accomplishments of giants of physics, in particular, Albert Einstein. On this suggestion, the author opted to purse studies in basic sciences instead of engineering sciences. In 2009, the author gave a lecture in Government College, Hyderabad on choice of gaussian surface based on symmetry considerations. Professor Mohummed Idrees Khan presented shield to the author after the lecture (Kamal, 2009*a*). The author's last face-to-face meeting with the late professor was in the year 2014 (Fig. 3). He spoke to the distinguished torch bearer of learning the last time via mobile phone during October 2018 to invite his mentor in a function at his residence. His last words to the author were, 'I am proud of you'. May Allah *Izz-o-Jal* give the author strength and capability to become the professor's ideal and shower His blessings on the grave of the deceased.

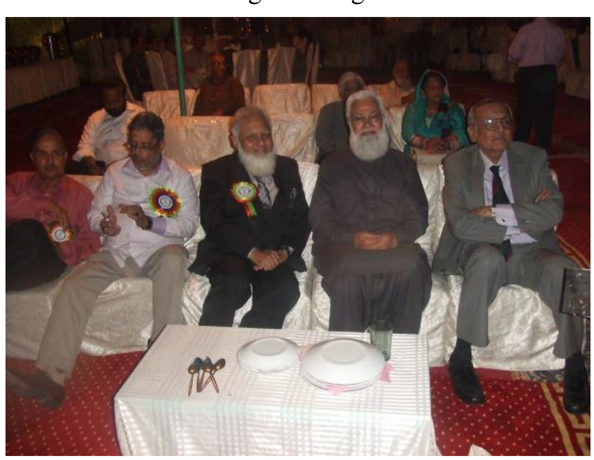

Fig. 3. Professor Mohummed Idrees Khan (2nd from right) during the 2014 Annual Function of Government College, Hyderabad, Sindh, Pakistan — the author seated on his right side

ENDNOTES

[1]Mathematical relationships, which define prepubertal, peripubertal and pubertal as well as their association with approximate Tanner scores is available in Kamal *et al.* (2017).

[2]Definition of sensitivity and specificity as well as relative sensitivity and relative specificity is available in Kamal *et al.* (2013*b*).

[3]The word *Asr* is an Arabic word, meaning frame; the name ‘Asr angle’ was given by the author (Kamal, 1982*a*; 1983*a*; *b*), motivated by verse (*ayah*) 28, chapter (*suratul dahr/insaǎn*) 76 of the Holy Quran**:**

نَحْنُ خَلَقْنٰهُمْ وَشَدَدْنَآ أَسْرَهُمْ ۚ وَإِذَا شِئْنَا بَدَّلْنَآ
أَمْثَالَهُمْ تَبْدِيلًا ﴿٢٨﴾

‘We, even We created them, and strengthened their frame. And when We will, We can replace them, bringing others like them in their steed’.

[4]The word ‘mesh’ is used in the spirit that the three unit vectors along the *x* axis, the *y* axis and the *z* axis, respectively, are related through the orthonormality conditions.

[5]*Guarded* means strict surveillance for possible harmful effects during stretching and *graduated* means systematic increase in time of stretching in order to condition the body to absorb the stress.

[6]A new branch of mathematics, *Condensed-Matter Mathematics*, was introduced in this lecture.

[7]Levi-Civita symbol is not a *tensor*, but a *tensor density*

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